Analytic electrical-conductivity tensor of a nondegenerate Lorentz plasma

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We have developed explicit quantum-mechanical expressions for the conductivity and resistivity tensors of a Lorentz plasma in a magnetic field. The expressions are based on a solution to the Boltzmann equation that is exact when the electric field is weak, the electron-Fermi-degeneracy parameter $\Theta \gg 1$, and the electron-ion Coulomb-coupling parameter $\Gamma/Z \ll 1$. (Γ is the ion-ion coupling parameter and Z is the ion charge state.) Assuming a screened $1/r$ electron-ion scattering potential, we calculate the Coulomb logarithm in the second Born approximation. The ratio of the term obtained in the second approximation to that obtained in the first is used to define the parameter regime over which the calculation is valid. We find that the accuracy of the approximation is determined by Γ/Z and not simply the temperature, and that a quantum-mechanical description can be required at temperatures orders of magnitude less than assumed by Spitzer [*Physics of Fully Ionized Gases* (Wiley, New York, 1962)]. When the magnetic field $\mathbf{B}=\mathbf{0}$, the conductivity is identical to the Spitzer result except the Coulomb logarithm $\ln \Lambda_1 = (\ln \chi_1 - \frac{1}{2}) + [(2Ze^2/\lambda m_e v_{e1}^2)(\ln \chi_1 - \ln 2^{4/3})]$, where $\chi_1 \equiv 2 m_e v_{e1} \lambda/\hbar$, m_e is the electron mass, $v_{e1} \equiv (7 k_B T/m_e)^{1/2}$, k_B is the Boltzmann constant, T is the temperature, λ is the screening length, $\hbar$ is Planck's constant divided by 2π, and e is the absolute value of the electron charge. When the plasma Debye length λ_D is greater than the ion-sphere radius a, we assume $\lambda = \lambda_D$; otherwise we set $\lambda = a$. The $\mathbf{B}=\mathbf{0}$ conductivity is consistent with measurements when $Z \gtrsim 1$, $\Theta \gtrsim 2$, and $\Gamma/Z \lesssim 1$, and in this parameter regime appears to be more accurate than previous analytic models. The minimum value of $\ln \Lambda_1$ when $Z \geq 1$, $\Theta \geq 2$, and $\Gamma/Z \leq 1$ is 1.9. The expression obtained for the resistivity tensor ($\mathbf{B} \neq \mathbf{0}$) predicts that $\eta_\perp/\eta_\parallel$ (where $\eta_\perp$ and $\eta_\parallel$ are the resistivities perpendicular and parallel to the magnetic field) can be as much as 40% less than previous analytic calculations. The results are applied to an idealized 17-MA z pinch at stagnation.

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I. INTRODUCTION

Electrical conduction in a plasma has been an active subject of theoretical research for over 50 years. In pioneering studies, Lorentz [1], Chapman, Enskog, and Cowling [2], Marshak [3], Cowling [4], Landshoff [5,6], Conwell and Weisskopf [7], Cohen, Spitzer, and Routly [8], Spitzer [9,10], and Spitzer and Härm [11] developed classical expressions for the electrical conductivity. These seminal results have been generalized by various methods [12–92]. In the presence of a magnetic field, the conductivity is a second-rank tensor; calculations of the tensor elements are presented in Refs. [2,4,5,9,10,12–17,19–21,25–27,30,31,33–35,38–41,43,45,52,55,57,58,61,74,77,80–82,87,88].

The conductivity is a function of momentum transfer in scattering events between electrons and other particles. Much of the early work assumes that electrons follow classical scattering trajectories. For electron-ion scattering, the classical Coulomb logarithm is a function of the electron impact parameter that results in an electron deflection of 90° [2,8,10,11]. Quantum-scattering effects are often introduced by correcting the classical result: the 90° impact parameter p_{90} is replaced by the electron de Broglie wavelength multiplied by a constant when the product is greater than p_{90} [5,8,10,15,26,40,57]. (Since the de Broglie wavelength $\propto v_e^{-1}$ where v_e is the electron speed, and $p_{90} \propto v_e^{-2}$, quantum effects become important at high temperature.) In such conductivity models the transition from classical to quantum scattering occurs between $(2.7\times 10^3)Z^2$ and $(5.8\times 10^5)Z^2$ K, depending on the constant assumed, where Z is the ionization charge state [5,8,10,15,26,40,57].

To include electron-diffraction effects in a self-consistent manner, a number of quantum-mechanical calculations have been performed. These studies often expand the electron distribution function in terms of Sonine (i.e., generalized Laguerre) polynomials. Kivelson and Dubois [24] obtain the conductivity from a one-Sonine-polynomial solution to quantum kinetic equations; Williams and DeWitt [36] present one-, two-, and three-Sonine-polynomial results. Boercker, Rogers, and DeWitt [50] use a correlation-function expression for the collision frequency to develop a one-Sonine conductivity. Ichimaru and Tanaka [59] and Kitamura and Ichimaru [78] obtain a one-Sonine solution by considering the scattering of electrons in the random potential fields of the ions. When the one-Sonine expressions [24,36,50,59,78] are corrected as described by Landshoff [5,6] to produce infinite-Sonine results, they are, in the high-temperature low-density limit, identical, and are dependent on scattering calculations performed in the first Born approximation.

We use here a more direct approach [1,3,7,12,13,57] that can be applied to a Lorentz plasma. The method is analytic and self-consistently includes quantum-scattering effects. Following Lorentz [1], Marshak [3], Conwell and Weisskopf [7], Brooks and Herring [12,13], and Lee and More [57], we develop in Sec. II an exact solution to the linearized Boltzmann transport equation for a nonrelativistic nondegenerate plasma in a weak electric field. We show that the Lorentz model, which assumes the ions are infinitely massive, is applicable to plasmas with finite-mass ions when the electron and ion temperatures are approximately equal. We assume electrons are scattered in binary collisions by a statically screened Coulomb potential, and that the screening length equals the ion-sphere radius when ion-ion coupling is strong. The Coulomb logarithm is obtained in the second Born approximation. We take into account the energy dependence of the Coulomb logarithm and evaluate it in closed form. The conductivity thus obtained differs from previous analytic results [1–18,20,21,23,24,26,29–31,33,35–41,43,46,47,49,53,54,56,57,59,61,62,64,66–73,76–78,80,81,86,87], and complements numerical quantum-mechanical calculations that are valid over a wider range of plasma parameters [21,23,25,28–30,32,33,36,42,44–60,62,63,65,66,68,69,71–81,83–92].

In Sec. III we present explicit analytic expressions for the conductivity and resistivity tensors of a plasma with a density gradient and a magnetic field. Because we calculate the Coulomb logarithm in the second Born approximation and take its energy dependence into account, we obtain results that differ from previous analytic expressions. (The results presented here neglect magnetic-field effects on the electron-ion scattering cross section; such effects are addressed by Daybelge [34], Yakovlev [52], Hernquist [55], Potekhin and Yakovlev [80,81,87,88], and Geller and Weisheit [82].)

In Sec. IV we show that the validity of the Boltzmann collision term, the scattering-potential model described in Sec. II, and the Coulomb logarithm are dependent on the electron-ion Coulomb-coupling parameter. Assuming the Coulomb logarithm is valid when the contribution from the second Born approximation is small, we find that a quantum-mechanical description can be accurate at temperatures orders of magnitude less than assumed previously [5,8,10,15,26,36,37,40,57].

In Sec. V we compare the results of Sec. II with the Spitzer conductivity model, the more-general quantum-mechanical conductivity model developed by Potekhin, Baiko, Haensel, Yakovlev, and Kaminker [80,81,83,86–88], and measurements performed on shock-heated xenon [93,94] and ohmically heated aluminum [95–97]. We demonstrate that there is no choice for the transition temperature in the Spitzer model that can bring it into agreement with the measurements. The results also appear inconsistent with discussions in Refs. [66,69,89,90], which suggest that for a singly ionized plasma the Born approximation is not valid for temperatures much less than 1.7×10^5 K. We find instead that the accuracy of the Born approximation is determined by the electron-ion Coulomb-coupling parameter, and not just the temperature. An example of the applicability of the results is given in Sec. VI, where we estimate the resistance of an idealized 17-MA tungsten z pinch at stagnation.

II. THEORETICAL SCALAR CONDUCTIVITY (B=0)

We consider a nonrelativistic two-component plasma consisting of electrons and ions at the same temperature T. We assume the electrons are nondegenerate; i.e., the electron thermal energy is much greater than the Fermi energy E_F,

$$\Theta \equiv \frac{k_B T}{E_F} = \frac{2 m_e k_B T}{\hbar^2 (3\pi^2 n_e)^{2/3}} = \frac{2 m_e k_B T}{\hbar^2 (3\pi^2 Z n_i)^{2/3}} \gg 1, \qquad (1)$$

where Θ is the electron-Fermi-degeneracy parameter [51,78], k_B is the Boltzmann constant, m_e is the electron mass, $\hbar$ is Planck's constant divided by 2π, n_e is the electron number density, n_i is the ion number density, and $Z \equiv n_e/n_i$ is the ionization charge state. (Equations are in cgs-Gaussian units throughout.) At an electron density of 10^{21} cm^{-3}, $E_F = 0.36$ eV (4.2×10^3 K).

To calculate the conductivity, we determine the steady-state current density as a function of electric field in the weak-field limit. The current density $\mathbf{j}=\mathbf{j}_e+\mathbf{j}_i$ where $\mathbf{j}_e$ and $\mathbf{j}_i$ are the electron and ion current densities, respectively. We evaluate the conductivity in the reference frame where the ion-fluid velocity equals zero; hence $\mathbf{j}_i=\mathbf{0}$ and $\mathbf{j}=\mathbf{j}_e$. The current densities and field can be readily transformed to other frames. The results developed in this section are, of course, directly applicable when $\mathbf{j}_i \ll \mathbf{j}_e$. This is a reasonable approximation, for example, in the frame where the total fluid momentum equals zero, since in this case $|\mathbf{j}_i| = (Z m_e/m_i)|\mathbf{j}_e| \ll |\mathbf{j}_e|$. The current density $\mathbf{j}=\mathbf{j}_e$ is calculated from the electron distribution function f_e.

We assume f_e satisfies the Boltzmann transport equation [98–100],

$$\frac{\partial f_e}{\partial t} + \mathbf{v}_e \cdot \frac{\partial f_e}{\partial \mathbf{r}} - \frac{e}{m_e} \mathbf{E} \cdot \frac{\partial f_e}{\partial \mathbf{v}_e} = \left.\frac{\partial f_e}{\partial t}\right|_{\text{collisions}}, \qquad (2)$$

where $f_e = f_e(t,\mathbf{r},\mathbf{v}_e)$, e is the absolute value of the electron charge, and $\mathbf{E}$ is the electric field. Without loss of generality, we assume $\mathbf{E}=E_z\mathbf{e}_z$ where $\mathbf{e}_z$ is the unit vector in the z direction. For the discussion in this section we assume that the magnetic field equals zero. The expression on the right-hand side of Eq. (2) is the collision term, i.e., the time rate of change of f_e due to collisions.

For a two-component plasma, the collision term is the sum of two expressions: one due to electron-electron collisions and the other to electron-ion collisions [98,100]. In this paper we assume the plasma is Lorentzian; i.e. that electron-electron collisions can be neglected and the ion mass $m_i \gg m_e$ [1,2,5,6,8,10,11,101]. (As described by Blatt [102] and others, the electron-fluid momentum does not change in an electron-electron collision. However, the electron velocity is randomized, which increases the electron-ion scattering probability and decreases the conductivity.) The effect of electron-electron collisions on the conductivity is a function of Z and Θ. In the nondegenerate limit ($\Theta \gg 1$), electron-electron collisions reduce the conductivity by 42% when $Z=1$; when $Z=16$, the reduction is 8% [5,6,10,11,61]. Due to the Pauli exclusion principle, these collisions become less significant as Θ is decreased [53,67,73,89,90]. According to Ref. [90], when $Z=1$, $T=10^4$ K, and $n_e=10^{19}$ cm^{-3} ($\Theta=51$), electron-electron collisions reduce the conductivity by ~20%; when $Z=1$, $T=10^4$ K, and $n_e=10^{21}$ cm^{-3} ($\Theta=2.4$), they have less than a 3% effect. Hence electron-electron collisions can be neglected when either $Z \gg 1$ or $\Theta \ll 50$.

Considering only electron-ion collisions, the collision term becomes [100]

$$\left.\frac{\partial f_e}{\partial t}\right|_{\text{collisions}} = \int\int [f_e(\mathbf{v}_e')f_i(\mathbf{v}_i') - f_e(\mathbf{v}_e)f_i(\mathbf{v}_i)]g\frac{\partial \sigma_{ei}}{\partial \Omega}d\Omega\, d\mathbf{v}_i, \tag{3}$$

where f_i is the ion distribution function, $\mathbf{v}_e$ and $\mathbf{v}_e'$ are the electron velocities before and after an electron-ion collision, $\mathbf{v}_i$ and $\mathbf{v}_i'$ are the corresponding ion velocities, $g \equiv |\mathbf{g}| \equiv |\mathbf{v}_e - \mathbf{v}_i|$, and $\partial\sigma_{ei}/\partial\Omega$ is the differential cross section for electron-ion scattering from $\mathbf{g}$ to $\mathbf{g}'$. The differential solid angle $d\Omega \equiv \sin\vartheta d\vartheta d\varphi$, where ϑ is the angle between $\mathbf{g}$ and $\mathbf{g}'$, and φ is the azimuthal scattering angle. The integration is over all angles and velocities. Equation (3) assumes elastic scattering (i.e., $|\mathbf{g}| = |\mathbf{g}'|$) and that there is only one ion species. (Generalizing to several ion species is straightforward [53,100].)

Because we assume that the electron and ion temperatures are equal, $m_e|\mathbf{v}_e|^2 = m_i|\mathbf{v}_i|^2$ for characteristic values of the electron and ion velocities. Hence

$$|\mathbf{v}_i| \ll |\mathbf{v}_e| \ll |\mathbf{v}_i|\frac{m_i}{m_e}. \tag{4}$$

Since [as indicated by Eq. (4)] the ion momentum is typically much greater than that of an electron, we can assume $f_i(\mathbf{v}_i') \approx f_i(\mathbf{v}_i)$. Since the characteristic electron speed is much greater than the ion speed, $g \approx v_e \equiv |v_e|$. Consequently Eq. (3) can be simplified as

$$\left.\frac{\partial f_e}{\partial t}\right|_{\text{collisions}} = n_i\int [f_e(\mathbf{v}_e') - f_e(\mathbf{v}_e)]v_e\frac{\partial\sigma_{ei}}{\partial\Omega}d\Omega. \tag{5}$$

Equation (5) is identical to that developed by Lorentz [1], Marshak [3], and Conwell and Weisskopf [7], who assume infinite-mass ions. Since Eq. (4) is most correct when $|\mathbf{v}_e| = |\mathbf{v}_i|(m_i/m_e)^{1/2}$, Eq. (5) is most applicable to plasmas with finite-mass ions when the electron and ion temperatures are approximately equal.

We assume that the electric field is sufficiently weak that f_e is only slightly perturbed from its equilibrium value. Hence we look for a solution to Eqs. (2) and (5) of the form

$$f_e = f_{e0} - \tau(v_e)\left.\frac{\partial f_e}{\partial t}\right|_{\text{collisions}} \equiv f_{e0} + f_{e1}, \tag{6}$$

where

$$f_{e0} \equiv n_e\left(\frac{m_e}{2\pi k_B T}\right)^{3/2}\exp\left(\frac{-m_e(v_{ex}^2+v_{ey}^2+v_{ez}^2)}{2k_B T}\right) \tag{7}$$

is the Maxwell-Boltzmann distribution function, $|f_{e1}| \ll f_{e0}$, and the velocity components in Eq. (7) are defined by $\mathbf{v}_e = v_{ex}\mathbf{e}_x + v_{ey}\mathbf{e}_y + v_{ez}\mathbf{e}_z$. ($\mathbf{e}_x$, $\mathbf{e}_y$, and $\mathbf{e}_z$ are unit vectors in the x, y, and z directions.) The relaxation time τ is assumed to be a function of the absolute magnitude but not the direction of $\mathbf{v}_e$, and is to be determined from Eqs. (2) and (5)–(7).

When $(\partial f_e/\partial t) = (\partial f_e/\partial \mathbf{r}) = 0$, Eqs. (2), (6), and (7) give

$$f_e = f_{e0} + \frac{e\tau}{m_e}E_z\frac{\partial f_{e0}}{\partial v_{ez}} = f_{e0} - \frac{e\tau}{k_B T}E_z f_{e0} v_{ez}. \tag{8}$$

Combining Eqs. (5)–(8), noting that $|\mathbf{v}_e| = |\mathbf{v}_e'|$, and using the relation [1,3]

$$1 - \frac{v_{ez}'}{v_{ez}} = 1 - \cos\vartheta - \sin\vartheta\cos\varphi\tan\psi, \tag{9}$$

where ψ is the angle between $\mathbf{v}_e$ and $\mathbf{e}_z$, we obtain

$$\tau(v_e) = \frac{1}{n_i v_e Q_{ei}} \tag{10}$$

$$Q_{ei} \equiv \int_0^{2\pi}\int_0^{\pi}(1-\cos\vartheta)\frac{\partial\sigma_{ei}}{\partial\Omega}\sin\vartheta d\vartheta\, d\varphi. \tag{11}$$

(Without loss of generality, we have chosen φ to be the angle between the plane formed by $\mathbf{v}_e$ and $\mathbf{v}_e'$ and the plane formed by $\mathbf{v}_e$ and $\mathbf{e}_z$ [1,3].) Q_{ei} is the cross section for momentum transfer from electrons to ions. Equations (7), (8), (10), and (11) are an exact solution to the Boltzmann equation for a nondegenerate Lorentz plasma in the weak-electric-field limit. The solution is identical to that developed by Lorentz [1], Marshak [3], and Conwell and Weisskopf [7], and used by Brooks and Herring [12,13] and Lee and More [57].

Defining $\mathbf{j} = \mathbf{j}_e = j_{ez}\mathbf{e}_z$ to be the current density in the direction of the field, we have

$$j_{ez} = -e\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_e v_{ez} dv_{ex} dv_{ey} dv_{ez}. \tag{12}$$

Combining Eqs. (7), (8), (10), and (12) we find

$$j_{ez} = \sigma E_z, \tag{13}$$

where

$$\sigma \equiv \frac{e^2}{n_i k_B T}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{f_{e0}}{v_e Q_{ei}}v_{ez}^2 dv_{ex} dv_{ey} dv_{ez}. \tag{14}$$

The electron-ion interactions are accounted for in the momentum-transfer cross section Q_{ei}.

To calculate Q_{ei}, we assume each electron is scattered by a screened Coulomb potential centered on a fixed ion,

$$V(r) = -\frac{Ze^2}{r}\exp\left(\frac{-r}{\lambda}\right). \tag{15}$$

Here r is the distance from the ion scattering center and λ is a screening length. Following Refs. [57,74,103,104] we set

$$\lambda = \lambda_D \equiv \left[\left(\frac{4\pi n_e e^2}{k_B T}\right) + \left(\frac{4\pi n_i Z^2 e^2}{k_B T}\right)\right]^{-1/2} \quad \text{when } \lambda_D \geq a, \tag{16}$$

$$\lambda = a \equiv \left(\frac{4}{3}\pi n_i\right)^{-1/3} \quad \text{when} \quad \lambda_D \leqslant a. \tag{17}$$

λ_D is the plasma Debye length and a is the ion-sphere radius.

As demonstrated in Refs. [28,51,103,105–109], the assumption of Debye screening becomes invalid when the plasma ions are strongly coupled; i.e., when the ion-ion Coulomb-coupling parameter $\Gamma > 1$, where

$$\Gamma \equiv \frac{(Ze)^2}{a k_B T} = \left(\frac{4}{3}\pi n_i\right)^{1/3} \frac{(Ze)^2}{k_B T}. \tag{18}$$

Γ is the ratio of the characteristic ion-ion Coulomb-interaction energy to the ion thermal energy [28,51,74,78,105]. Equations (16)–(18) can be combined to give

$$\Gamma = \frac{Z}{3(Z+1)}\left(\frac{a}{\lambda_D}\right)^2. \tag{19}$$

Hence $\lambda_D = a$ in the transition region between weak and strong coupling [28,105].

Assuming the electron-ion interaction potential given by Eq. (15), Dalitz [110] calculates the quantum-mechanical scattering amplitude of a nonrelativistic electron in the second Born approximation. Using this result, we obtain the differential electron-ion scattering cross section $\partial\sigma_{ei}/\partial\Omega$ to order $Z^3 e^6$,

$$\frac{\partial\sigma_{ei}}{\partial\Omega} = \frac{Z^2 e^4 \chi^4}{4 m_e^2 v_e^4 (1+\chi^2\alpha^2)^2} + \left[\frac{Z^3 e^6 \chi^4}{\hbar m_e^2 v_e^5 \alpha (1+\chi^2\alpha^2)(4+4\chi^2+\chi^4\alpha^2)^{1/2}} \arctan \frac{\chi\alpha}{(4+4\chi^2+\chi^4\alpha^2)^{1/2}}\right], \tag{20}$$

where

$$\chi \equiv 2k\lambda, \tag{21}$$

$$k \equiv \frac{m_e v_e}{\hbar}, \tag{22}$$

$$\alpha \equiv \sin\frac{\vartheta}{2}, \tag{23}$$

and λ is the screening distance defined by Eq. (15). The reduced mass is approximated as m_e. (The differential cross section $\partial\sigma_{ei}/\partial\Omega$ is also given in Ref. [111]; however, the sign before the second term on the right-hand side of Eq. (10.136) in Ref. [111] is incorrect. We expect on physical grounds that the second term on the right-hand side of Eq. (20) is positive for an attractive potential, and negative when the potential is repulsive.) Equation (20) is consistent with scattering amplitudes presented in Refs. [37,111–115]. It appears that the result obtained in the first Born approximation [the first term on the right-hand side of Eq. (20)] was originally given by Wentzel [116].

Equation (20) can be simplified by noticing that the arctangent argument has a maximum value of $8^{-1/2}$ at $\chi = 2^{1/2}$ and $\alpha = 1$ [37]. Hence to within an error $\leqslant 4\%$ we can write

$$\arctan \frac{\chi\alpha}{[4+4\chi^2+\chi^4\alpha^2]^{1/2}} \approx \frac{\chi\alpha}{[4+4\chi^2+\chi^4\alpha^2]^{1/2}}. \tag{24}$$

Furthermore, as shown at the end of this section, the assumption $\Theta \gg 1$ [Eq. (1)] implies that characteristic values of χ are much greater than 1. When $\chi \geqslant 10$, the error introduced by Eq. (24) is $\leqslant 0.3\%$.

Combining Eqs. (11) and (20)–(24), and making the substitution $\sin^2(\vartheta/2) = (1-\cos\vartheta)/2$, gives [117]

$$Q_{ei} = \frac{4\pi Z^2 e^4}{m_e^2 v_e^4}\left[\ln(1+\chi^2)^{1/2} - \left(\frac{1}{2}\right)\left(\frac{\chi^2}{1+\chi^2}\right)\right] + \frac{8\pi Z^3 e^6}{\lambda m_e^3 v_e^6}\left[\frac{(4+4\chi^2)}{(4+3\chi^2)} \ln\left(\frac{4+4\chi^2+\chi^4}{4+4\chi^2}\right)^{1/2} - \frac{\chi^2 \ln(1+\chi^2)^{1/2}}{(4+3\chi^2)}\right]. \tag{25}$$

(When $Z=1$ and $\lambda = \lambda_D$, Eq. (25) is identical to Eq. (6.11) in Ref. [54] except the sign before the term proportional to e^6 in Ref. [54] is incorrect. Q_{ei} is given to order e^8 in Eq. (7.64) of Ref. [62]; however, this expression also has the incorrect sign before the e^6 term; in addition, the e^8 term does not include one of the two contributions obtained in the second Born approximation, and the contribution obtained in the third.)

Since (as we shall show) $\chi \gg 1$, we can express Eq. (25) as

$$Q_{ei} = \frac{4\pi Z^2 e^4}{m_e^2 v_e^4} \ln\Lambda(v_e), \tag{26}$$

where

$$\ln\Lambda(v_e) \equiv (\ln\chi - \tfrac{1}{2}) + \left[\frac{2Ze^2}{\lambda m_e v_e^2}(\ln\chi - \ln 2^{4/3})\right]. \tag{27}$$

The error due to approximating Eq. (25) as Eqs. (26) and (27) is less than 1% when $\chi \geqslant 10$ and $(2Ze^2/\lambda m_e v_e^2) \leqslant 0.5$. (The quantity $2Ze^2/\lambda m_e v_e^2$ is the ratio of the characteristic electron-ion potential energy to the electron kinetic energy.) Equation (27) is consistent with the approximation for Q_{ei} given in Ref. [37].

Combining Eqs. (7), (14), and (26) gives the electrical conductivity:

$$\sigma = \frac{m_e^{7/2}}{3(2\pi)^{3/2} Z e^2 (k_B T)^{5/2}} \int_0^\infty \frac{v_e^7}{\ln \Lambda(v_e)} \exp\left(\frac{-m_e v_e^2}{2k_B T}\right) dv_e . \tag{28}$$

To evaluate this integral, we note that the function $v_e^7 \exp(-m_e v_e^2/2k_B T)$ is peaked, and find that its maximum value occurs at $v_{e1} \equiv (7k_B T/m_e)^{1/2}$. [There appears to be a misprint in Ref. [7], where this speed is given as $(6k_B T/m_e)^{1/2}$.] We use here the subscript 1 because other values of the electron speed, labeled 2 and 3, are defined in Sec. III. When $\chi = 2k_1\lambda \gg 1$, where $k_1 \equiv m_e v_{e1}/\hbar$, the function $\ln \Lambda(v_e)$ is slowly varying in the vicinity of v_{e1}, i.e., over the interval of v_e that contributes most to the integral. In this limit we can approximate σ by assuming that $\ln \Lambda(v_e)$ is constant and equal to its value at $v_e = v_{e1}$ [7], which we label as $\ln \Lambda_1$. Hence

$$\sigma = \frac{2(2k_B T)^{3/2}}{\pi^{3/2} Z e^2 m_e^{1/2} \ln \Lambda_1}, \tag{29}$$

where

$$\ln \Lambda_1 \equiv (\ln \chi_1 - \tfrac{1}{2}) + \left[\frac{2Ze^2}{\lambda m_e v_{e1}^2} (\ln \chi_1 - \ln 2^{4/3})\right], \tag{30}$$

$$\chi_1 \equiv 2k_1 \lambda \equiv \frac{2 m_e v_{e1} \lambda}{\hbar}, \tag{31}$$

$$v_{e1} \equiv \left(\frac{7k_B T}{m_e}\right)^{1/2}. \tag{32}$$

The two terms on the right-hand side of Eq. (30) are obtained in the first and second Born approximations, respectively. Equation (29) is identical to the conductivity of a nondegenerate Lorentz plasma given in Refs. [2,3,5–7,10–13,15,26,57,86], except that the Coulomb logarithm defined by Eqs. (30)–(32) differs from those obtained previously.

It is often convenient to express the quantities $\ln \Lambda_1$ and χ_1 in terms of Z, Γ, and Θ. When $\Gamma \leqslant Z/3(Z+1)$, then $\lambda = \lambda_D$ and we have

$$\ln \Lambda_1 = (\ln \chi_1 - \tfrac{1}{2}) + \left[\frac{[12(Z+1)\Gamma^3]^{1/2}}{7Z^{3/2}} (\ln \chi_1 - \ln 2^{4/3})\right], \tag{33}$$

$$\chi_1 = 2k_1 \lambda_D = \left(\frac{7^3 3\pi^2}{2}\right)^{1/6} \left(\frac{Z^{5/3}}{Z+1}\right)^{1/2} \left(\frac{\Theta}{\Gamma}\right)^{1/2}. \tag{34}$$

When $\Gamma \geqslant Z/3(Z+1)$, then $\lambda = a$ and

$$\ln \Lambda_1 = (\ln \chi_1 - \tfrac{1}{2}) + \left[\frac{2\Gamma}{7Z} (\ln \chi_1 - \ln 2^{4/3})\right], \tag{35}$$

$$\chi_1 = 2k_1 a = \left(\frac{7^3 3^4 \pi^2}{2}\right)^{1/6} Z^{1/3} \Theta^{1/2}. \tag{36}$$

As asserted earlier, we see from Eqs. (34) and (36) that assuming $\Theta \gg 1$ implies $\chi_1 \gg 1$, whether $\lambda = \lambda_D$ or $\lambda = a$. (When $\Theta \geqslant 2$, $\chi_1 \geqslant 10.2$.)

III. THEORETICAL CONDUCTIVITY TENSOR ($\mathbf{B} \neq 0$)

The results of the preceding section can be generalized to obtain explicit analytic expressions for the electrical conductivity and resistivity tensors of a plasma with a density gradient and a magnetic field.

We again work in the reference frame where the ion-fluid velocity equals zero. The results given in Secs. III A and III B are also directly applicable when $|\mathbf{j}_i| \ll |\mathbf{j}_e|$, which is a good approximation in the frame where the total fluid momentum equals zero. This is often the case in a steady-state system. However, the results of these sections must be used with care. For example, we consider a plasma with no pressure gradients, **E** perpendicular to **B**, and electron and ion collision frequencies much less than the electron and ion cyclotron frequencies, respectively. In such a system the electrons and ions $\mathbf{E} \times \mathbf{B}$ drift at the same velocity, and $\mathbf{j}_i = -\mathbf{j}_e$. In the frame where the ion-fluid velocity equals zero, $\mathbf{j}_i = \mathbf{j}_e = \mathbf{0}$ and $\mathbf{E} = \mathbf{0}$ [10,26].

A. Weak magnetic field ($\omega_{ce}\tau \lesssim 1$)

Using ideas developed in Sec. II and by Lorentz [1], Marshak [3], Kittel [17], Kubo [27], and Lee and More [57], we obtain, after some algebra [118], the following results in the weak-electric-field limit. Without loss of generality we assume the magnetic field $\mathbf{B} = B_z \mathbf{e}_z$. We express the results in terms of a generalized electric field $\mathbf{E}_g$ [4,19], which we define in Eq. (48). We assume that the direction of $\mathbf{E}_g$ with respect to **B** is arbitrary, so that $\mathbf{E}_g = E_{gx}\mathbf{e}_x + E_{gy}\mathbf{e}_y + E_{gz}\mathbf{e}_z$.

We find [118] that the second-rank conductivity tensor $\underline{\sigma}$ is given by

$$\mathbf{j}_e = \underline{\sigma} \mathbf{E}_g, \tag{37}$$

where

$$\underline{\sigma} \equiv \begin{pmatrix} \sigma_\perp & -\sigma_\wedge & 0 \\ \sigma_\wedge & \sigma_\perp & 0 \\ 0 & 0 & \sigma_\parallel \end{pmatrix}, \tag{38}$$

$$\sigma_\perp \equiv \frac{\sigma}{1+\omega_{ce}^2 \tau_1^2}, \tag{39}$$

$$\omega_{ce} \equiv \frac{eB_z}{m_e c}, \tag{40}$$

$$\tau_1 \equiv \tau(v_{e1}) = \frac{m_e^2 v_{e1}^3}{4\pi n_i Z^2 e^4 \ln \Lambda_1}, \tag{41}$$

$$\sigma_\wedge \equiv \frac{\beta \omega_{ce} \tau_2 \sigma}{1+\omega_{ce}^2 \tau_2^2}, \tag{42}$$

$$\beta \equiv \frac{63\pi^{1/2}\ln\Lambda_1}{2^6 5^{1/2}\ln\Lambda_2}, \tag{43}$$

$$\ln\Lambda_2 \equiv \ln\Lambda(v_{e2}), \tag{44}$$

$$v_{e2} \equiv \left(\frac{10k_BT}{m_e}\right)^{1/2}, \tag{45}$$

$$\tau_2 \equiv \tau(v_{e2}) = \frac{m_e^2 v_{e2}^3}{4\pi n_i Z^2 e^4 \ln\Lambda_2}, \tag{46}$$

$$\sigma_\parallel \equiv \sigma, \tag{47}$$

$$\mathbf{E}_g \equiv \mathbf{E} + \frac{k_BT}{n_e e}\left(\frac{\partial n_e}{\partial \mathbf{r}}\right). \tag{48}$$

The quantities σ, $\ln\Lambda_1$, and v_{e1} are given by Eqs. (29), (30), and (32), respectively; τ_1 and τ_2 are defined using Eqs. (10) and (26); and $\ln\Lambda_2$ is defined using Eq. (27).

The above expression for $\underline{\boldsymbol{\sigma}}$ is similar in form to that given by Urpin and Yakovlev [45], Adamyan [77], and others. $\sigma_\perp$ is the conductivity associated with current flowing perpendicular to **B** and parallel to $\mathbf{E}_g$, $\sigma_\wedge$ with current flowing perpendicular to both **B** and $\mathbf{E}_g$, and $\sigma_\parallel$ with current flowing parallel to **B**. We find $\sigma_\parallel$ is independent of **B**, as expected. Equations (39) and (42) are valid when $\omega_{ce}\tau_1$, $\omega_{ce}\tau_2 \lesssim 1$ [118].

The second-rank resistivity tensor $\underline{\boldsymbol{\eta}}$ is defined by

$$\underline{\boldsymbol{\eta}}\mathbf{j}_e = \mathbf{E}_g, \tag{49}$$

where

$$\underline{\boldsymbol{\eta}} \equiv \underline{\boldsymbol{\sigma}}^{-1} \equiv \begin{pmatrix} \eta_\perp & \eta_\wedge & 0 \\ -\eta_\wedge & \eta_\perp & 0 \\ 0 & 0 & \eta_\parallel \end{pmatrix} \tag{50}$$

$$\equiv \begin{pmatrix} \dfrac{\sigma_\perp}{\sigma_\perp^2+\sigma_\wedge^2} & \dfrac{\sigma_\wedge}{\sigma_\perp^2+\sigma_\wedge^2} & 0 \\ \dfrac{-\sigma_\wedge}{\sigma_\perp^2+\sigma_\wedge^2} & \dfrac{\sigma_\perp}{\sigma_\perp^2+\sigma_\wedge^2} & 0 \\ 0 & 0 & \dfrac{1}{\sigma_\parallel} \end{pmatrix}. \tag{51}$$

In the limit $\omega_{ce}\tau_1$, $\omega_{ce}\tau_2 \to 0$, we have from Eqs. (39)–(47), (50), and (51) (to first order in B_z)

$$\eta_\perp = \frac{1}{\sigma}, \tag{52}$$

$$\eta_\wedge = \frac{315\pi}{512}\left(\frac{\ln\Lambda_1}{\ln\Lambda_2}\right)^2 \frac{B_z}{n_e ec}, \tag{53}$$

$$\eta_\parallel = \frac{1}{\sigma}. \tag{54}$$

The transverse resistivity $\eta_\wedge$ differs by the factor $(\ln\Lambda_1/\ln\Lambda_2)^2$ from the results given in Refs. [26,57] for a Lorentz plasma, since in the previous work the Coulomb logarithms are evaluated at the speed corresponding to the average electron thermal energy.

B. Strong magnetic field ($\omega_{ce}\tau \gg 1$)

When $\omega_{ce}\tau(v_e) \gg 1$ for characteristic values of v_e, we find that the conductivity tensor elements $\sigma_\perp$, $\sigma_\wedge$, and $\sigma_\parallel$ can be expressed as follows [118]:

$$\sigma_\perp = \left(\frac{\ln\Lambda_1}{48\ln\Lambda_3}\right)\frac{\sigma}{\omega_{ce}^2\tau_3^2}, \tag{55}$$

$$\ln\Lambda_3 \equiv \ln\Lambda(v_{e3}), \tag{56}$$

$$v_{e3} \equiv \left(\frac{k_BT}{m_e}\right)^{1/2}, \tag{57}$$

$$\tau_3 \equiv \tau(v_{e3}) = \frac{m_e^2 v_{e3}^3}{4\pi n_i Z^2 e^4 \ln\Lambda_3}, \tag{58}$$

$$\sigma_\wedge = \frac{n_e ec}{B_z}, \tag{59}$$

$$\sigma_\parallel = \sigma. \tag{60}$$

The quantities σ and $\ln\Lambda_1$ are given by Eqs. (29) and (30), respectively; $\ln\Lambda_3$ is defined using Eq. (27); and τ_3 is defined using Eqs. (10) and (26). Equations (55) and (59) are applicable when $\omega_{ce}\tau_3 \gg 1$ [118].

The resistivity tensor $\underline{\boldsymbol{\eta}}$ is obtained from Eqs. (50), (51), and (55)–(60). In the limit $\omega_{ce}\tau_3 \to \infty$, we have

$$\eta_\perp = \frac{32}{3\pi}\left(\frac{\ln\Lambda_3}{\ln\Lambda_1}\right)\frac{1}{\sigma}, \tag{61}$$

$$\eta_\wedge = \frac{B_z}{n_e ec}, \tag{62}$$

$$\eta_\parallel = \frac{1}{\sigma}. \tag{63}$$

The ratio $\eta_\perp/\eta_\parallel$ differs by the factor $(\ln\Lambda_3/\ln\Lambda_1)$ from the results given in Refs. [10,26,57] for a Lorentz plasma, since in the previous work the Coulomb logarithms are evaluated at the speed corresponding to the average electron thermal energy.

IV. LIMITS OF APPLICABILITY

The results of Secs. II and III assume the Maxwell-Boltzmann distribution function, the Boltzmann collision term, the scattering-potential model described in Sec. II, and the Coulomb logarithm given by Eq. (27). They also assume that the integration of Eq. (28) described in Sec. II, and similar integrations required to develop Eqs. (39), (42), (55), and (59), are reasonably accurate. In this section we discuss the limits of applicability of these assumptions. In particular, we demonstrate that the classical distribution is valid to first order when $\Theta \geqslant 2$, and that the Boltzmann collision term, the scattering potential model, and the Coulomb logarithm require weak electron-ion coupling ($\Gamma/Z \ll 1$).

A. Maxwell-Boltzmann distribution

To estimate the range of temperatures over which the Maxwell-Boltzmann distribution is valid, we use the Fermi-Dirac distribution to calculate numerically the average electron kinetic energy as a function of Θ. When $\Theta = 10$ the average energy is within 1% of $3k_BT/2$, the Maxwellian result. When $\Theta = 2$ the average is 5% above the classical value; when $\Theta = 1$ the average is 13% higher. We conclude that the Maxwell-Boltzmann distribution is a reasonable approximation for Θ as low as 2. Since we assume the electrons are nonrelativistic, we also require that $k_BT \ll m_ec^2$ [i.e., $(\Gamma^2\Theta/Z^{10/3}) \gg (2^7/3^4\pi^2)^{1/3}(e^2/c\hbar)^2 \sim 3\times10^{-5}$].

The electron component of the screening distance given by Eq. (16) is the electron Debye length $\lambda_{D,e} \equiv (k_BT/4\pi n_e e^2)^{1/2}$. This expression assumes that the electrons are Maxwellian. Assuming instead the Thomas-Fermi potential and Fermi-Dirac distribution, we find that when $\Theta \geqslant 2$, the error in $\lambda_{D,e}$ is $\lesssim$5%. This introduces a $\lesssim$1% error in the Coulomb logarithm. Hence it appears that Eq. (16) is a valid approximation for Θ as low as 2.

B. Boltzmann collision term

Equation (3) assumes molecular chaos and that particles interact only through binary collisions [27,98–100,119]. We expect these assumptions to be most reasonable when the characteristic momentum-transfer cross section is much less than πa^2 [27,99,119],

$$Q_{ei}(v_{e1}) = \frac{4\pi Z^2 e^4 \ln\Lambda_1}{m_e^2 v_{e1}^4} \ll \pi a^2. \tag{64}$$

Expressed in terms of Γ, Eq. (64) becomes

$$\left(\frac{\Gamma}{Z}\right)^2 \ll \frac{49}{4\ln\Lambda_1}. \tag{65}$$

Equations (64) and (65) are identical to the condition that the mean free path $[n_iQ_{ei}(v_{e1})]^{-1} \gg 4a/3$. Γ/Z, the electron-ion Coulomb-coupling parameter, is the ratio of the characteristic electron-ion Coulomb-interaction energy to the electron thermal energy [105]. Hence it appears that the Boltzmann collision term is applicable when electron-ion coupling is weak.

For the molecular-chaos assumption to be applicable, we also require that ion positions be weakly correlated, i.e., that the ion structure factor be $\sim$1. We estimate the effect of ion correlations using the effective static structure factor $S''(q) = 1 - \exp(-13a^2q^2/3\Gamma)$ given in Refs. [86,87], where $q \equiv 2k\sin(\vartheta/2)$. Using Eqs. (11), (26), and the total scattering cross section obtained from Eq. (20), we find that when the plasma ions are strongly coupled ($\lambda = a$), the characteristic value of $a^2q^2 = 2\ln\Lambda_1$. Hence $S''(q) \sim 1$ when $\Gamma \ll 26(\ln\Lambda_1)/3$. Using the conductivity model developed in Refs. [80,81,83,86–88], we estimate that the error introduced in the conductivity by neglecting ion correlations is less than $\exp[-26(\ln\Lambda_1)/3\Gamma]$ when this quantity is much less than 1. When the plasma ions are weakly coupled ($\lambda = \lambda_D$), then to an extremely good approximation $S''(q) = 1$.

C. Scattering-potential model

The scattering potential model defined by Eqs. (15)–(17) assumes that when ion-ion coupling is strong, the effective screening length $\lambda = a$ [Eq. (17)]. Calculations by Hubbard [28,44,58] show that this simple model leads to an error in the Coulomb logarithm $\ln\Lambda_1$ of at most -10%, $+12\%$ when $\Gamma/Z = 1$, $\Theta = 2$, and $1 \leqslant \Gamma \leqslant 5$. The error is reduced as Γ/Z is decreased and Θ increased. At larger values of Γ, the screening length becomes significantly less than a; at $\Gamma = 40$ the value consistent with Hubbard's calculations is $0.7a$ [28,44,58].

Equation (17) also assumes that the electrons form a uniform background of negative charge that neutralizes the ion space charge [28,51,57,74,103–109]. This is valid when the characteristic electron-ion Coulomb-interaction energy is much less than the average electron thermal energy [105],

$$\frac{Ze^2}{a} \ll \frac{3k_BT}{2}. \tag{66}$$

Expressed in terms of the electron-ion coupling parameter Γ/Z, Eq. (66) becomes [105]

$$\frac{\Gamma}{Z} \ll \frac{3}{2}. \tag{67}$$

[Equations (66) and (67) are identical to the condition $(\lambda_{D,e}/a)^2 \gg 2/9$, which is given in terms of the electron Debye length.] A uniform electron background can also be assumed when

$$Z^{1/3}\Theta^{1/2} \lesssim \left(\frac{2^{13}\pi^4}{3^7}\right)^{1/6}, \tag{68}$$

i.e., the electron thermal de Broglie wavelength $h/(3m_ek_BT)^{1/2} \gtrsim a$.

In addition, Eqs. (15)–(17) implicitly assume that the magnetic field does not affect the electron-ion scattering cross section. This is valid when both $\lambda \ll (3k_BT/m_e)^{1/2}(m_ec/eB)$ (i.e., the screening distance λ is much less than the electron Larmor radius [34,82]), and $k_BT \gg \hbar\omega_{ce}$ (the magnetic field is nonquantizing [52,55,80,81,87,88]).

D. Coulomb logarithm

As discussed in Sec. I, it has been assumed by several authors that a transition takes place between classical and quantum-mechanical scattering at temperatures that range from $(2.7\times10^3)Z^2$ to $(5.8\times10^5)Z^2$ K [5,8,10,15,26,36,37,40,57]. To determine in a more quantitative manner when a quantum-mechanical description is required, we consider Eq. (30). The first term on the right-hand side is obtained in the first Born approximation; the term proportional to Ze^2 is obtained in the second. We assume that the accuracy of this equation is given by the ratio δ of these terms [111],

$$\delta \equiv \left(\frac{2Ze^2}{\lambda m_e v_{e1}^2} \right) \left(\frac{\ln \chi_1 - \ln 2^{4/3}}{\ln \chi_1 - \frac{1}{2}} \right). \tag{69}$$

Since $\chi_1 \gg 1$, δ is approximately equal to the ratio of the characteristic electron-ion potential energy to the electron kinetic energy when $v_e = v_{e1}$. The ratio δ is reduced as λ is increased (which is counterintuitive since the scattering potential increases everywhere as λ increases). Hence it appears that Eq. (30) is most valid in the high-temperature low-density limit, i.e., when $\lambda \to \infty$.

We do not prove that $\delta \ll 1$ is a sufficient condition for the validity of Eq. (30), but propose this as a plausible hypothesis. We show in Sec. V that this assumption appears consistent with experimental results, and is more accurate than the quantum-mechanical model presented in Refs. [5,8,10,15,26,40,57]. We also note that (as is well known) when $\lambda \to \infty$, $\partial \sigma_{ei} / \partial \Omega$ [Eq. (20)] approaches the exact nonrelativistic quantum-mechanical Rutherford cross section obtained by Gordon [120].

When $\Gamma \leqslant Z/3(Z+1)$ (i.e., when ion-ion coupling is weak), then $\lambda = \lambda_D$ and δ is obtained from Eq. (33),

$$\delta = \frac{[12(Z+1)]^{1/2} \Gamma^{3/2}}{7Z^{3/2}} \left(\frac{\ln \chi_1 - \ln 2^{4/3}}{\ln \chi_1 - \frac{1}{2}} \right). \tag{70}$$

In this case it appears that the Born approximation is always a reasonable estimate: since the maximum value of $\Gamma = Z/3(Z+1)$, δ never exceeds 5%. When $\Gamma \geqslant Z/3(Z+1)$ (ion-ion coupling is strong), then $\lambda = a$, δ is obtained from Eq. (35),

$$\delta = \frac{2\Gamma}{7Z} \left(\frac{\ln \chi_1 - \ln 2^{4/3}}{\ln \chi_1 - \frac{1}{2}} \right), \tag{71}$$

and we expect the Born approximation to be valid when

$$\frac{\Gamma}{Z} \ll \frac{7}{2}, \tag{72}$$

i.e., when electron-ion Coulomb coupling is weak. (As noted earlier, the conditions given by Eqs. (65), (67), and (72) are consistent.)

The discrepancy between Eqs. (69)–(71) and assumptions made previously [5,8,10,15,26,36,37,40,57] can be significant. We consider, for example, a plasma with $Z = 10$, $\Gamma = 5$, and $\Theta = 50$. The temperature of such a system is 3.0×10^5 K. For these conditions $\delta = 0.13$, which suggests that the Born approximation would be applicable. However, according to Refs. [8,10], electron-ion scattering in such a plasma does not require a quantum-mechanical treatment for temperatures as high as $(4.2 \times 10^5) Z^2 = 4.2 \times 10^7$ K. Hence, assuming the Born approximation is valid when $\delta \ll 1$, we find that electron-ion scattering can be quantum mechanical at temperatures orders of magnitude less than previously given. (This is discussed further in Sec. V A.)

E. Integration over electron speed

The integration of Eq. (28) discussed in Sec. II assumes $\ln \Lambda(v_e)$ is constant. To estimate the resulting error, we integrate Eq. (28) numerically using Eq. (27) for $\ln \Lambda(v_e)$, from $v_{e(\min)}$ to ∞, where $v_{e(\min)}$ is the minimum speed for which Eq. (27) is estimated to be valid. [We cannot integrate Eq. (28) numerically from 0 to ∞ since Eq. (27) is developed in the Born approximation, and as discussed above, appears to be valid only when $2Ze^2/\lambda m_e v_e^2 \ll 1$. The contribution to the integral from 0 to $v_{e(\min)}$ is negligible if $v_{e(\min)} \ll v_{e1}$ and $\ln \Lambda(v_e)$ is reasonably behaved.] When $\Gamma / Z \leqslant 1$, $\Theta \geqslant 2$, and $v_{e(\min)} = v_{e1}/2$, the numerical result agrees with Eqs. (29)–(32) to within $\leqslant 3\%$. Similar considerations apply to the calculations leading to Eqs. (39), (42), (55), and (59).

V. RESULTS

A. B=0

In Tables I and II we compare the results of Sec. II to other theoretical predictions and to conductivity measurements. Because the effects of electron-electron collisions are expected to be significant for some of the plasmas considered, we correct the conductivity given in Sec. II for electron-electron scattering, and give corrected values in the tables. We define the corrected conductivity as

$$\sigma_C \equiv \gamma \sigma, \tag{73}$$

where σ is given by Eq. (29). Following Refs. [67,73], we make the simplifying assumption that γ is a function only of Z and Θ (i.e., we neglect the dependence of γ on Γ). We approximate γ as

$$\gamma = \gamma_\infty + \frac{(1-\gamma_\infty)}{\left[1 + 0.6 \ln\left(\frac{\Theta}{20} + 1 \right) \right]} \tag{74}$$

$$\gamma_\infty \equiv \frac{3\pi}{32} \left(1 + \frac{6.453Z^2 + 14.669Z}{2.697Z^2 + 11.615Z + 7.645} \right). \tag{75}$$

Equation (74) was developed empirically to be consistent with the theoretical results shown in Fig. 4 of Ref. [90]; the agreement is to within $\pm 2\%$ for $2.4 \leqslant \Theta \leqslant 1100$. Comparing Eq. (74) with other calculations [62], we estimate the uncertainty in γ to be $\sim 10\%$. Equation (75) is taken from Ref. [61] and is consistent to within 1% with results presented in Refs. [5,6,10,11].

We include in the tables a corrected Spitzer-Härm conductivity that we define as

$$\sigma_{SHC} \equiv \gamma \sigma_{SH} \equiv \gamma \frac{2(2k_B T)^{3/2}}{\pi^{3/2} Z e^2 m_e^{1/2} \ln \Lambda_{SH}}, \tag{76}$$

$$\ln \Lambda_{SH} \equiv \ln \left[1 + \left(\frac{\lambda_D}{p_{\min}} \right)^2 \right]^{1/2}, \tag{77}$$

TABLE I. Comparison of measured conductivities of shock heated xenon with theoretical predictions. The measurements were performed by Mintsev, Fortov, Pavlov, and Gryaznov [93,94]. The values of σ_{SHC}, σ_{MSHC}, σ_{PC}, and σ_C are given by Eqs. (76), (82), (84), and (73), respectively. [To convert from cgs-Gaussian units (statohm cm)$^{-1}$ to $(\Omega\,\text{cm})^{-1}$, the theoretical conductivities have been multiplied by 1.113×10^{-12}.] The ionization charge states listed in the first column were inferred from the measurements [94] and are used for the conductivity calculations. The quantity δ [Eq. (69)] is the ratio of the Coulomb-logarithm term obtained in the second approximation to that obtained in the first. The correction for electron-electron scattering γ [Eq. (74)] ranges from 0.90 to 0.97 for the plasmas listed.

Z	T (10^3 K)	n_e (cm^{-3})	Γ	Γ/Z	Θ	δ	σ_{expt} $(\Omega\,\text{cm})^{-1}$	σ_{SHC} $(\Omega\,\text{cm})^{-1}$	σ_{MSHC} $(\Omega\,\text{cm})^{-1}$	σ_{PC} $(\Omega\,\text{cm})^{-1}$	σ_C $(\Omega\,\text{cm})^{-1}$
1.8	47	4×10^{20}	1.12	0.62	20.5	0.15	470 ± 190	1600	845	522	366
2.4	70	6×10^{20}	1.40	0.58	23.3	0.15	700 ± 280	2170	1110	676	484
2.8	95	6×10^{20}	1.33	0.48	31.6	0.12	550 ± 220	2350	1340	830	628
2.1	65	1.0×10^{21}	1.43	0.68	15.4	0.17	620 ± 250	2720	1270	788	532
1.9	62	1.3×10^{21}	1.38	0.73	12.3	0.18	670 ± 270	3040	1370	876	573
2.2	70	1.5×10^{21}	1.64	0.75	12.6	0.18	700 ± 280	3490	1450	895	582
1.5	64	2.0×10^{21}	1.04	0.69	9.53	0.17	750 ± 300	3480	1780	1265	830
1.3	50	3.4×10^{21}	1.25	0.96	5.23	0.23	830 ± 330	4820	1820	1326	738
2.2	76	2.4×10^{21}	1.77	0.80	10.0	0.20	1100 ± 440	4570	1740	1089	684
1.6	59	4.3×10^{21}	1.63	1.02	5.27	0.24	1300 ± 520	6080	1990	1363	740
1.1	44	4.5×10^{21}	1.18	1.08	3.81	0.25	970 ± 390	5570	1960	1541	784
0.9	37	6.0×10^{21}	1.11	1.23	2.65	0.27	730 ± 290	6610	2090	1845	824
0.7	29	4.8×10^{21}	0.865	1.24	2.41	0.27	680 ± 270	5460	1870	1824	792

where

$$p_{\min}=p_{\text{classical}} \quad \text{when} \quad p_{\text{classical}}\geqslant p_{\text{qm}}, \tag{78}$$

$$p_{\min}=p_{\text{qm}} \quad \text{when} \quad p_{\text{classical}}\leqslant p_{\text{qm}}, \tag{79}$$

$$p_{\text{classical}}\equiv\frac{Ze^2}{3k_BT}, \tag{80}$$

$$p_{\text{qm}}\equiv\frac{\hbar}{2(3m_ek_BT)^{1/2}}\equiv\zeta\frac{h}{(3m_ek_BT)^{1/2}}. \tag{81}$$

σ_{SH} is the Lorentz-plasma conductivity given in Refs. [8,10,11]; γ is defined by Eq. (74); λ_D is the Debye length given by Eq. (16); $\zeta\equiv1/4\pi$; and h is Planck's constant. The quantity $p_{\text{classical}}\equiv p_{90}[v_e=(3k_BT/m_e)^{1/2}]$, where p_{90} is the classical impact parameter that results in a 90° deflection for an electron with speed v_e [2,8,10,11]. Hence $p_{\text{classical}}$ is equal to p_{90} at the speed $v_e=(3k_BT/m_e)^{1/2}$ that corresponds to the average electron thermal energy. The quantity $h/(3m_ek_BT)^{1/2}$ in Eq. (81) is the electron thermal de Broglie wavelength.

Since Refs. [8,10,11] assume $\lambda_D/p_{\min}\gg1$, the Coulomb logarithm given in these References is approximated as $\ln(\lambda_D/p_{\min})$. Equations (77), (78), and (80) give the exact expression [2] for classical scattering by an unscreened Coulomb potential assuming the maximum impact parameter equals λ_D, and that the effective classical value of $p_{\min}$ can be approximated as $p_{\text{classical}}$ [10]. The correction to $p_{\min}$ indicated by Eqs. (79) and (81) is introduced in Ref. [8,10] to account heuristically for quantum effects. According to Eqs. (78)–(81), the transition from classical to quantum-mechanical scattering occurs at a temperature of $(4.2\times10^5)Z^2$ K [8,10]. Other authors have used this model [Eqs. (78)–(81)] with different values for the constant ζ [5,15,26,40,57]; in these discussions, the transition temperature varies from $(2.7\times10^3)Z^2$ to $(5.8\times10^5)Z^2$ K.

TABLE II. Comparison of measured conductivities of ohmically heated aluminum with theoretical predictions. The measurements were performed by Benage, Shanahan, and Murillo [95–97]. The values of σ_{SHC}, σ_{MSHC}, σ_{PC}, and σ_C are given by Eqs. (76), (82), (84), and (73), respectively. [To convert from cgs-Gaussian units (statohm cm)$^{-1}$ to $(\Omega\,\text{cm})^{-1}$, the theoretical conductivities have been multiplied by 1.113×10^{-12}.] The ionization charge states listed in the first column were inferred from the measurements [97] and are used for the conductivity calculations. The quantity δ [Eq. (69)] is the ratio of the Coulomb-logarithm term obtained in the second approximation to that obtained in the first. The correction for electron-electron scattering γ [Eq. (74)] ranges from 0.94 and 0.98 for the plasmas listed.

Z	T (10^3 K)	n_e (cm^{-3})	Γ	Γ/Z	Θ	δ	σ_{expt} $(\Omega\,\text{cm})^{-1}$	σ_{SHC} $(\Omega\,\text{cm})^{-1}$	σ_{MSHC} $(\Omega\,\text{cm})^{-1}$	σ_{PC} $(\Omega\,\text{cm})^{-1}$	σ_C $(\Omega\,\text{cm})^{-1}$
2.2	104	3.00×10^{22}	3.00	1.36	2.55	0.31	2000 ± 300	24900	4450	3130	1360
2.5	146	2.01×10^{22}	2.31	0.92	4.67	0.22	1495 ± 220	15500	4660	3266	1830
3.1	201	1.18×10^{22}	2.01	0.65	9.16	0.16	1370 ± 210	11200	4700	3206	2130
3.8	253	7.89×10^{21}	1.96	0.52	15.1	0.13	1545 ± 230	9720	4690	3131	2250
4.2	285	6.47×10^{21}	1.93	0.46	19.4	0.11	2000 ± 300	9230	4740	3137	2340

We also include in the tables a modified Spitzer-Härm conductivity that we define as

$$\sigma_{MSHC} \equiv \gamma \sigma_{MSH} \equiv \gamma \frac{2(2k_B T)^{3/2}}{\pi^{3/2} Z e^2 m_e^{1/2} \ln \Lambda_{MSH}}, \tag{82}$$

$$\ln \Lambda_{MSH} \equiv \ln \left[1 + \left(\frac{\lambda}{p_{\min}} \right)^2 \right]^{1/2}. \tag{83}$$

σ_{MSHC} is identical to σ_{SHC} except that Eqs. (76) and (77) assume that the screening distance λ always equals λ_D, whereas Eqs. (82) and (83) assume λ is defined by Eqs. (16) and (17).

In addition, we list in Tables I and II the conductivity calculated by Potekhin, Baiko, Haensel, Yakovlev, and Kaminker [80,81,83,86–88]. This model was developed primarily to calculate transport coefficients of degenerate relativistic electrons in neutron-star envelopes, and according to Ref. [88] provides order-of-magnitude estimates in the nondegenerate regime. We define σ_{PC} as

$$\sigma_{PC} \equiv \gamma \sigma_P, \tag{84}$$

where σ_P is calculated using the FORTRAN source code available from Ref. [88].

The conductivity measurements listed in Table I were performed on shock-heated xenon by Mintsev, Fortov, Pavlov, and Gryaznov [93,94]. The experimental error is estimated as 30–50 % [94]; a value of 40% is used for the errors listed in the table. The measurements in Table II were performed on ohmically-heated aluminum by Benage, Shanahan, and Murillo [95–97]. The experimental error for these measurements is estimated as 15% [95–97]. The charge states given in Tables I and II were inferred from the measurements [94,97] and are used for the calculations of σ_{SHC}, σ_{MSHC}, σ_{PC}, and σ_C.

There appears to be a large discrepancy between the measurements and σ_{SHC}. There is better agreement with σ_{MSHC}, which suggests that the scattering-potential model given by Eqs. (16) and (17) is more accurate than simply assuming $\lambda = \lambda_D$. [For all of the measurements listed in Tables I and II, $\Gamma > Z/3(Z+1)$; hence all of the values of σ_{MSHC} and σ_C listed assume $\lambda = a$.] Comparing σ_{MSHC} and σ_C with $\sigma_{\rm expt}$ we see that agreement with experiment is better for σ_C, which suggests the Born approximation is more accurate than the heuristic quantum-mechanical model given by Eqs. (78)–(81).

According to Eqs. (78)–(81), the Coulomb logarithm can be calculated classically for all the plasmas listed in Tables I and II [8,10,26]. Assuming $\delta \ll 1$ is a sufficient condition for the validity of Eq. (30), we find instead that the Coulomb logarithm is quantum mechanical for all of the results in these tables, at temperatures as much as a factor of 35 less than the transition between classical and quantum scattering estimated in Refs. [8,10,26]. (As discussed in Sec. IV, this discrepancy can, for other plasma conditions, exceed two orders of magnitude.) The model given by Eqs. (78)–(81) cannot be improved simply by changing the constant ζ in Eq. (81), as is done in Refs. [5,15,40,57]: increasing ζ would either increase or leave unchanged each of the values of σ_{SHC} and σ_{MSHC} listed in the tables; decreasing ζ would have no effect.

These results appear inconsistent with the discussion in Refs. [66,69,89,90], which suggests that when $Z=1$, $\Gamma<1$, and $\Theta>1$, the Born approximation is not applicable when $T \ll 1.7\times10^5$ K (i.e., $\Gamma^2\Theta \gg 1$), and that at these temperatures a quasiclassical calculation is required. We find instead that when $Z=1$, the Born approximation is valid over the entire parameter regime defined by $\Gamma \lesssim 1$ and $\Theta \gtrsim 2$, and not just when $T > 1.7\times10^5$ K. However, the resulting discrepancy in the conductivity is not great, which indicates there is a large range over which the quasiclassical and Born calculations are both valid. For example, when $Z=1$, $T=10^4$ K, and $n_e = 1\times10^{17}$ cm^{-3} ($\Gamma=0.125$, $\Theta=1100$), the contribution to the Coulomb logarithm from the second Born approximation is 3%; hence we expect the approximation to be valid. Neglecting electron-electron collisions, we find that for this plasma the conductivity [Eq. (29)] is 50 $(\Omega\,\text{cm})^{-1}$; the model described in Refs. [66,69,89,90] obtains 63 $(\Omega\,\text{cm})^{-1}$. The discrepancy is reduced as Γ is decreased.

Comparing σ_{PC} [Eq. (84)] and σ_C with $\sigma_{\rm expt}$ in Tables I and II, we see that σ_C appears, on average, to be in somewhat better agreement with experiment. [This assumes Eq. (74) is an accurate correction for electron-electron scattering.] For the 18 measurements listed in these tables, the square root of the average value of $[(\sigma_{PC}-\sigma_{\rm expt})/\sigma_{\rm expt}]^2$ is 81% ; for σ_C, this quantity is 28%. When $\Gamma/Z \lesssim 1$ and $\Theta \gtrsim 2$, there are four significant differences between σ_{PC} and σ_C. The first is that σ_{PC} is calculated in the first Born approximation whereas σ_C is calculated in the second. The quantity δ [Eq. (69)], the ratio of the Coulomb-logarithm term obtained in the second approximation to that obtained in the first, is listed in Tables I and II, and for the plasmas considered is found to be as large as 30%.

The second difference is due to the scattering potentials assumed by the two models. The model described in Sec. II [Eqs. (16) and (17)] finds that for all the plasmas listed in Tables I and II, $\lambda = a$. This appears to be more accurate than the model assumed in Refs. [80,81,83,86–88], which finds that for the plasmas listed the effective screening distance is significantly less than a. The third difference is that we assume a Maxwell-Boltzmann distribution for the electrons, whereas Refs. [80,81,83,86–88] assume the more-accurate Fermi-Dirac distribution. The fourth is that we integrate Eq. (28) analytically as described in Sec. II, whereas in Refs. [80,81,83,86–88] the calculations are performed numerically. The third and fourth differences together have a 4% effect on the calculated conductivity for the first plasma listed in Table II, and a 1–3 % effect for the other entries in Tables I and II.

We find that even though the Ioffe Institute model [80,81,83,86–88] was developed primarily for degenerate systems, it is significantly more accurate than any possible version of the Spitzer-Härm conductivity for the nondegenerate plasmas listed in Tables I and II. We also note that the Ioffe model is considerably more accurate than stated in Ref. [88], and that it would be straightforward to improve its accuracy for nondegenerate systems.

TABLE III. Calculated elements of the resistivity tensor for a Lorentz plasma. The Coulomb logarithms $\ln\Lambda_B$ and $\ln\Lambda_{LM}$ are defined in Refs. [26] and [57], respectively; $\ln\Lambda_1$, $\ln\Lambda_2$, and $\ln\Lambda_3$, are defined by Eqs. (30), (44), and (56). The quantity η_0 is given by Eq. (90). Without loss of generality we have assumed $\mathbf{B}=B_z\mathbf{e}_z$. In the high-temperature low-density limit, the quantities $\ln\Lambda_B$, $\ln\Lambda_{LM}$, $\ln\Lambda_1$, $\ln\Lambda_2$, and $\ln\Lambda_3$, converge to the same value.

	Braginskii [26]	Lee and More [57]	Secs. II and III
$\eta_\parallel$	$\eta_0\ln\Lambda_B$	$\eta_0\ln\Lambda_{LM}$	$\eta_0\ln\Lambda_1$
$\frac{\eta_\perp}{\eta_\parallel}(B_z\to 0)$	1	1	1
$\eta_\wedge(B_z\to 0)$	$\frac{1.98B_z}{n_eec}$	$\frac{1.9328B_z}{n_eec}$	$\frac{315\pi}{512}\left(\frac{\ln\Lambda_1}{\ln\Lambda_2}\right)^2\frac{B_z}{n_eec}$
$\frac{\eta_\perp}{\eta_\parallel}(B_z\to\infty)$	$\frac{32}{3\pi}$	3.32	$\frac{32}{3\pi}\left(\frac{\ln\Lambda_3}{\ln\Lambda_1}\right)$
$\eta_\wedge(B_z\to\infty)$	$\frac{B_z}{n_eec}$	$\frac{B_z}{n_eec}$	$\frac{B_z}{n_eec}$

Assuming a 10% uncertainty in each of the quantities Z, T, n_e, γ, and σ, we find that the conductivity σ_C is consistent (to within the error in $\sigma_{\rm expt}$) with all of the measurements in Table I, and three of the five in Table II. Because σ_C is in reasonable agreement with most of the measurements, we expect it to provide a useful analytic estimate for the conductivity of a Lorentz plasma when $Z\gtrsim 1$, $\Gamma/Z\lesssim 1$, and $\Theta\gtrsim 2$. The minimum value of the Coulomb logarithm $\ln\Lambda_1$ [Eq. (30)] when $Z\geqslant 1$, $\Gamma/Z\leqslant 1$, and $\Theta\geqslant 2$ is 1.9. (The minimum is achieved when $Z=1$, $\Gamma=1/6$, and $\Theta=2$.) Given the approximations and uncertainties involved, this is consistent with the assumption by Lee and More [57] that, in general, the minimum value of the Coulomb logarithm is 2.

We note some care must be taken when comparing conductivity models with each other and with experimental results. For example, for each of the conductivities $\sigma_{\rm expt}$ listed in Table I, the xenon shock velocity was measured to infer the xenon-plasma mass density and pressure. These, in turn, were used in an equation-of-state calculation to infer the temperature and ionization charge state. When the mass density and inferred temperature are subsequently input to a computational algorithm to determine a theoretical conductivity, the computation, if it includes a different equation-of-state calculation, may find a charge state that is not consistent with the one originally used to interpret the measurements. Comparisons between theory and experiment, and between different theoretical models are, of course, more meaningful when the same charge state is used throughout (as in Tables I and II), to eliminate discrepancies in the conductivity due simply to differences in the equations of state.

B. $\mathbf{B}\neq 0$

Braginskii [26] and Lee and More [57] give explicit expressions for the resistivity tensor of a Lorentz plasma. In Table III we compare these results in the limits $B_z\to 0$ and $B_z\to\infty$ to the predictions of Secs. II and III.

The Braginskii tensor elements can be expressed in the notation of Secs. II and III by combining Eqs. (2.2e) and (4.30) of Ref. [26]. Assuming the plasma is in a steady state, electron and ion pressures are scalars, and temperature gradients can be neglected, we find that (in the frame where the ion-fluid velocity equals zero) these equations give

$$\mathbf{E}_g=\eta_\perp\mathbf{j}_{e\perp}-\frac{\eta_\wedge}{B_z}\mathbf{B}\times\mathbf{j}_{e\perp}+\eta_\parallel\mathbf{j}_{e\parallel}, \tag{85}$$

where

$$\eta_\perp\equiv\left(1-\frac{4.63\omega_{ce}^2\tau_e^2+0.0678}{\omega_{ce}^4\tau_e^4+7.482\omega_{ce}^2\tau_e^2+0.0961}\right)\frac{m_e}{\tau_en_ee^2}, \tag{86}$$

$$\eta_\wedge\equiv\left(\frac{\omega_{ce}\tau_e(1.704\omega_{ce}^2\tau_e^2+0.0940)}{\omega_{ce}^4\tau_e^4+7.482\omega_{ce}^2\tau_e^2+0.0961}\right)\frac{m_e}{\tau_en_ee^2}+\frac{B_z}{n_eec}, \tag{87}$$

$$\eta_\parallel\equiv\left(\frac{3\pi}{32}\right)\frac{m_e}{\tau_en_ee^2}=\eta_0\ln\Lambda_B, \tag{88}$$

$$\tau_e\equiv\left(\frac{3\pi}{32}\right)\frac{m_e^{1/2}2(2k_BT)^{3/2}}{\pi^{3/2}Z^2e^4n_i\ln\Lambda_B}, \tag{89}$$

$$\eta_0\equiv\frac{\pi^{3/2}Ze^2m_e^{1/2}}{2(2k_BT)^{3/2}}. \tag{90}$$

Equations (86)–(90) assume the plasma is Lorentzian. (The calculations by Braginskii include the effects of electron-electron collisions and are more general than Eqs. (86)–(90). We present here only the results of Refs. [26] obtained for a Lorentz plasma.) References [15] and [26] use the notation $1/\sigma_\perp$ when $\eta_\perp$ is intended; Eqs. (85)–(90) have been rewritten to be consistent with the notation in this paper. The current densities $\mathbf{j}_{e\parallel}$ and $\mathbf{j}_{e\perp}$ are parallel and perpendicular to $\mathbf{B}$, respectively. The Coulomb logarithm $\ln\Lambda_B$ is that given by Braginskii [26]. (Reference [26] has two discussions on the transition temperature between classical and quantum scattering for the Coulomb logarithm. On page 215, this temperature is given as $5.8\times10^5\ Z^2$ K. On page 238, because of a typographical error, it is not clear whether $2.7\times10^3\ Z^2$ or $1.1\times10^5\ Z^2$ K is intended.) The quantity τ_e is the relaxation time defined by Eq. (2.5e) of Ref. [26].

The Lee-More results are summarized as Eqs. (52) and (62) in Ref. [57]. These are expressed in terms of the Hall coefficient, defined in Ref. [57] as $R\equiv\eta_\wedge/B$. (The Hall coefficient is normally defined as $-\eta_\wedge/B$ [77].) In the nondegenerate limit, we can express the Lee-More resistivity parallel to the magnetic field as $\eta_0 \ln\Lambda_{LM}$ where η_0 is defined by Eq. (90), and the Lee-More Coulomb logarithm $\ln\Lambda_{LM}$ is described in Ref. [57]. (We note that Ref. [57] contains several typographical errors and implicit assumptions. There appear to be sign errors in Eqs. (46) and in some of the previous expressions in Refs. [57]. The sign before the Hall coefficients R and $R_\perp$ in Eqs. (52) and (62), respectively, are incorrect. The superscripts ξ and η are interchanged in Eqs. (58a) and (58b), and also in Eqs. (B12) and (B13). The expression $1/\sigma_\perp$ is used when $\eta_\perp$ is intended. Equation (62) in Ref. [57] is not valid when the Larmor radius is less than the screening distance of the scattering potential, or when the magnetic field is quantizing, so this equation is not, in fact, valid at arbitrarily strong magnetic fields. Also, the results in Ref. [57] implicitly assume a frame of reference where the ion-fluid velocity equals zero. In addition, as discussed in Sec. II above, it appears that since the Lee-More model assumes a Lorentz plasma, it is most accurate when the electron and ion temperatures are approximately equal.)

Without loss of generality we assume $\mathbf{B}=B_z\mathbf{e}_z$ for the results listed in Table III. The predictions of Secs. II and III for $\eta_\perp/\eta_\parallel$ and $\eta_\wedge$ differ somewhat from the earlier calculations. For example, we consider a plasma with $Z=1$, $\Gamma=0.06$, and $\Theta=2$. Under these conditions, in the limit $B_z\to\infty$ ($B_z=5\times10^9$ G) we find that $\eta_\perp/\eta_\parallel=2.05$, which is 40% less than the value $32/3\pi=3.395$ predicted by Braginskii [26] and Lee and More [57]. The value $32/3\pi$ is also obtained by Spitzer [10]. Numerical calculations using the model presented in Refs. [80,81,83,86–88] find that for these conditions $\eta_\perp/\eta_\parallel=2.09$. When $Z=1$, $\Gamma=0.06$, $\Theta=2$, and $B_z\to0$, we find $(\eta_\wedge n_e ec/B_z)=1.68$, which is 15% and 13% less than the values given in Refs. [26] and [57], respectively. For these conditions, the model described in Refs. [80,81,83,86–88] gives $(\eta_\wedge n_e ec/B_z)=1.61$. In the high-temperature low-density limit ($\Gamma\ll1,\Theta\gg1$), the Coulomb logarithms $\ln\Lambda_B$, $\ln\Lambda_{LM}$, $\ln\Lambda_1$, $\ln\Lambda_2$, and $\ln\Lambda_3$ converge to the same value, and the quantities $\eta_\perp/\eta_\parallel$ and $\eta_\wedge$ as determined in Secs. II and III agree with the earlier analytic results [10,26,57].

VI. 17-MA TUNGSTEN *Z* PINCH

Experiments on the *Z* accelerator [121–129] are presently being conducted with a 5.9-mg tungsten-wire-array *z* pinch that is 1 cm in length and has a 2 cm initial diameter [130–134]. The measured pinch current rises to its peak value of 19 MA in 100 ns; the current at stagnation is ~17 MA. The resistance of the pinch which is of interest for electrical simulations of the pinch-accelerator system [122,128,129], can estimated using the results of Secs. II and III.

At stagnation, when the power radiated in x rays by the pinch reaches its peak value (130 TW [130–136]), we can model the pinch plasma as a 0.1-cm-radius, 1-cm-length cylinder. We assume the temperature and axial current density are uniform throughout, and that the electron and ion number densities are uniform in the axial direction. We estimate $T\sim2.5\times10^6$ K (210 eV), $n_i\sim6.2\times10^{20}\ \mathrm{cm}^{-3}$, and $Z\sim31$ [130–136]. Since $Z\gg1$, we can neglect the effects of electron-electron collisions on the conductivity [5,6,10,11,61]. The plasma is not degenerate ($\Theta=80$) and electron-ion coupling is fairly weak ($\Gamma/Z=0.3$), but ion-ion coupling is strong ($\Gamma=9$). (Hence the pinch is a nondegenerate Coulomb liquid.) We approximate the screening distance as $\lambda=a$ in accordance with Eq. (17).

We estimate the resistance in the direction of the pinch current, which we label as being in the *x* direction. We define the *z* direction to be that of the azimuthal magnetic field. Assuming that at stagnation the electron and ion currents in the *y* direction (the Hall currents) equal zero, we have from Eqs. (49) and (50) that $\eta_\perp j_{ex}=E_{gx}=E_x$. (We set $E_{gx}=E_x$ since we neglect density gradients in the *x* direction.) Since the magnetic field at the edge of the pinch is 3.4×10^7 G, characteristic values of $\omega_{ce}\tau_1$, $\omega_{ce}\tau_2\sim1-2$. Using Eqs. (39), (42), (50), and (51) we estimate that $\eta_\perp\sim2\times10^{-4}\ \Omega$ cm, which is three times as resistive as room-temperature stainless steel. The pinch resistance is ~7 mΩ. Since the current at stagnation is 17 MA, the resistive voltage drop across the pinch is 10^5 V. Similar calculations suggest that the resistance during most of the wire-array implosion is also on the order of a few milliohms. Assuming that these estimates adequately describe a real pinch, it appears that the performance of the *Z* accelerator is not significantly affected by the pinch resistance, since it is much less than the pinch impedance (~0.1–0.4 Ω) due to inductive effects.

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[118] Please see EPAPS Document No. [E-PLEEE8-66-086210] for a detailed calculation of the conductivity tensor elements. A direct link to this document may be found in the online article's HTML reference section. This document may be retrieved via the EPAPS homepage (http://www.aip.org/pubservs/epaps.html) or from ftp.aip.org in the directory /epaps/. Please see the EPAPS homepage for more information.
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